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Stabilization of some systems with constant delay

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Abstract: An algorithm for stabilization of a class of systems with constant delay is proposed.

Keywords: Stabilization, time delay, stability, differential equations.

1. INTRODUCTION

We consider control systems of differential equations with constant delay of the form

$$\begin{aligned} dx(t)/dt = {} & A_1(t)x(t) + B_1(t)x(t-\tau) + A_2(t)y(t) \\ & +B_2(t)y(t-\tau) + C_1u_1(t) \\ dy(t)/dt = {} & \vartheta_0 e^t[A_3(t)x(t) + B_3(t)x(t-\tau) \\ & +A_4(t)y(t) + B_4(t)y(t-\tau)] + C_2u_2(t), \end{aligned}$$

$$t \geq 0,\ \ \vartheta_0 = const,\ \ \vartheta_0 > 0,\ \ \tau = const,\ \ \tau > 0. \quad (1)$$

We assume that the matrices $A_j(t), B_j(t)$ $(j = 1, 2, \dots, 4)$ of the size $m\times m$ are periodic (with period τ), differentiable sufficient number of times; vector-functions $x(t), y(t)$ have dimension m. The solution $\{x(t), y(t)\}^\top$ is defined at the initial segment $[-\tau, 0]$ by the piecewise continuous vector function $\phi(\eta) = \{\phi_1(\eta), \phi_2(\eta)\}^\top$. The components $u_j(t)$ of control $u(t) = \{u_1(t), u_2(t)\}^\top$ are r-dimensional vector functions, the matrices C_j have dimensions $m \times r$.

The systems of this kind can be obtained from the systems with linear delay $(1 - \mu)\vartheta$ $(\mu = const, 0 < \mu < 1)$ by the substitution $t = \ln\frac{\vartheta}{\vartheta_0}$. Systems with linear delay occur in problems of mechanics, physics (Bellman, 1963, p. 96), biology, for example, in the study of oscillation process of the current collector of the moving locomotive at interaction with the contact wire (taking into account the impact of elastic supports) Fox (1971); Ockendon (1971). Let the solution of the system be unstable for $u(t) \equiv 0$. Consider the problem of stabilization of the system on infinite time interval.

2. EXPONENTIAL STABILITY CONDITIONS

Consider $u(t) \equiv 0$. We pass to the countable differential-difference system on the finite time interval $[0, \tau]$ (Bellman, 1963, p. 103). We have the relations

$$\begin{aligned} dx_{n+1}(t)/dt &= A_1(t)x_{n+1}(t) + B_1(t)x_n(t) \\ &\quad +A_2(t)y_{n+1}(t) + B_2(t)y_n(t) \\ \varepsilon_n dy_{n+1}(t)/d\tau &= e^t(A_3(t)x_{n+1}(t) + B_3(t)x_n(t) \\ &\quad +A_4(t)y_{n+1}(t) + B_4(t)y_n(t)),\ \ t \in [0, \tau], \end{aligned}$$

$$x_{n+1}(t) = x(t + n\tau),\ \ y_{n+1}(t) = y(t + n\tau),\ \ \varepsilon_n = \frac{\mu^n}{\vartheta_0} \quad (2)$$

with the boundary conditions

$$x_{n+1}(0) = x_n(\tau),\ \ y_{n+1}(0) = y_n(\tau).$$

The initial function vector is the expression $\{x_0(t), y_0(t)\}^\top$. We consider the linear normalized space R^m in which the norm of a vector $\bar{w} = \{\bar{w}_j\}^\top$ (here $\bar{w}_j$ $(j = 1, ..., m)$ are components of the vector $\bar{w}$, $\top$ is transpose icon) is defined, for example, as $\|\bar{w}\| = \sum_{j=1}^{m} |\bar{w}_j|$. We define the norm of the matrix $D = \{d_{ij}\}$ in accordance with the norm of the vector (Barbashin, 1967, p. 12): $\|D\| = \max_j \sum_i |d_{ij}|$. We also introduce the norm of a vector function $\bar{w}(t)$ on a segment $[0, \tau]$ as

$$\|\bar{w}(t)\|_\tau = \sup_{t\in[0,\tau]} \|\bar{w}(t)\|. \quad (3)$$

When using norm (3), the space of vector functions becomes a Banach space (Barbashin, 1967, p. 143), we denote it C_m. Consider asymptotic properties of the differential system (2) for small values of ε_n.

It is shown in Grebenshchikov (2017) that the first-order approximation of system (2) is the differential system

$$\begin{aligned} dx^0_{n+1}(t)/dt &= 0.5[(A^0_1 + B^0_1)x^0_{n+1}(t) \\ &\quad +A^0_2 y^0_{n+1}(t) + B^0_2 y^0_n(t)] \\ \varepsilon_n dy^0_{n+1}(t)/dt &= e^t[(A_3(t) + B_3(t))x^0_{n+1}(t) \\ &\quad +A_4(t)y^0_{n+1}(t) + B_4(t)y^0_n(t)], \end{aligned}$$

$$A^0_j = \frac{2}{\tau}\int_0^{2\tau} A_j(\xi)d\xi,\ \ B^0_j = \frac{2}{\tau}\int_0^{2\tau} B_j(\xi)d\xi,\ \ j = 1, 2. \quad (4)$$

This result is obtained by replacing the argument in the system: $t = \theta_n\varepsilon_n$ (Mitropolskii, 1971, p. 169). After performing this substitution in system (4) one can get

$$\begin{aligned} dx^0_{n+1}(\theta_n)/d\theta_n &= 0.5\varepsilon_n[(A^0_1 + B^0_1)x^0_{n+1}(\theta_n) \\ &\quad +A^0_2 y^0_{n+1}(\theta_n) + B^0_2 y^0_n(\theta_n)] \\ dy^0_{n+1}(\theta_n)/d\theta_n &= e^{\theta_n\varepsilon_n}[A^{\varepsilon_n}_3(\theta_n) + B^{\varepsilon_n}_3(\theta_n)x^0_{n+1}(\theta_n) \\ &\quad +A^{\varepsilon_n}_4(\theta_n)y^0_{n+1}(\theta_n) + B^{\varepsilon_n}_4(\theta_n)y^0_n(\theta_n)], \end{aligned}$$

$$A^{\varepsilon_n}_j(\theta_n) = A_j(\theta_n\varepsilon_n),\ \ B^{\varepsilon_n}_j(\theta_n) = B_j(\theta_n\varepsilon_n),\ j = 3, 4. \quad (5)$$

Since subsystems in system (5) may be weakly connected (Furasov, 1977, p. 113), we assume that for the matrices A^0_1, B^0_1 the following terms are valid:

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1) the eigenvalues λ of matrix A_1^0 have negative real part

$$Re(\lambda) < -\beta_1,\ \ \beta_1 = const,\ \ \beta_1 > 0; \qquad (6)$$

2) the eigenvalues $\bar{\lambda}$ of the matrix $A_1^0 + B_1^0$ also have a negative real part, i.e.

$$Re(\bar{\lambda}) < -2\beta_2,\ \ \beta_2 = const,\ \ \beta_2 > 0. \qquad (7)$$

For the second subsystem in (5) we suppose that for the eigenvalues $\hat{\lambda}(t)$ of the matrix $A_4(t)$ the following inequality is valid

$$Re(\hat{\lambda}(t)) < -\beta_3,\ \ \beta_3 = const,\ \ \beta_3 > 0,\ t \in [0, \tau] \qquad (8)$$

and in addition to this for the eigenvalues $\rho(t)$ of the matrix $-A_4^{-1}(t)B_4(t)$ the following inequality holds

$$|\rho(t)| < \gamma,\ \ \gamma = const,\ \ 0 < \gamma < 1,\ t \in [0, \tau]. \qquad (9)$$

From the first subsystem of system (5) due to the presence of the multiplier ε_n in the right-hand side, it follows that the value $x_{n+1}(t)$ satisfies the asymptotic equality

$$\begin{aligned} x_{n+1}(t) &= -2(A_1^0 + B_1^0)^{-1}(A_2^0 y_{n+1}^0(t) + B_2^0 y_n^0(t)) \\ &+\mathbf{O}(\|x_{n+1}(t)\|_\tau + \|y_{n+1}(t)\|_\tau + \|y_n(t)\|_\tau). \qquad (10) \end{aligned}$$

It is known (Grebenshchikov (2017, 2012)) that due to inequalities (6)-(9) weakly connected system is exponentially stable. Here is an example where these conditions are not sufficient for exponential stability of system (5). Consider the fourth order system

$$\begin{aligned} dx_1(t)/dt &= -(1 + \cos(2\pi t))x_1(t) + \sin(2\pi t)x_1(t-1) \\ &\quad + \cos(2\pi t)x_2(t) \\ dx_2(t)/dt &= -x_2(t) + 0.25\cos(2\pi t)x_2(t-1) + \delta_1 y_2(t) \\ dy_1(t)/dt &= e^t[(2\cos(2\pi t) - 1)y_1(t) - 2(1 - \sin(2\pi t))y_2(t) \\ &\quad +(0.5 + \cos(2\pi t))y_1(t-1) \\ &\quad +0.5(1 - \sin(2\pi t))y_2(t-1)] \\ dy_2(t)/dt &= e^t[2(1 + \sin(2\pi t))y_1(t) - (1 - 2\cos(2\pi t))y_2(t) \\ &\quad -(1 + \sin(2\pi t))y_1(t-1) \\ &\quad +0.25(1 - 2\cos(2\pi t))y_2(t-1) + \delta_2 x_2(t)], \\ &\quad \delta_j = const,\ \ \delta_j \neq 0,\ \ t \geq 0. \qquad (11) \end{aligned}$$

Let the values δ_j be small enough. If we consider now the first-order approximation system, we obtain the relations

$$A_1^0 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix},\ \ \lambda_1 = \lambda_2 = -1,\ \ B_1^0 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix},$$

$$A_4(t)^{-1}B_4(t) = \begin{pmatrix} -0.5 & 0 \\ 0 & -0.25 \end{pmatrix},$$

in this case the matrix $A_4(t)$ is periodic, the period is 1, it has eigenvalues $\hat{\lambda}_1(t) = \hat{\lambda}_2(t) = -1$, the matrix $B_4(t)$ is also periodic, it's period is equal to 1, the absolute values of the eigenvalues of the matrix $-A_4(t)^{-1}B_4(t)$ $\rho_1 = -0.5$, $\rho_2 = -0.25$ are less than 1, i.e. for a sufficiently small δ_j $(j = 1, 2)$ system (11) is exponentially stable. It is illustrated by the graph of the solution shown on Fig.1 for the initial vector function $\phi(\xi) = \{1; 1\}^\top$.

If the values of δ_j are not small enough, the system can be unstable as it can be seen from the graph of the solution shown on Fig. 2. So, there is a problem of stabilization of system (2).

3. BUILDING AN ALGORITHM OF STABILIZATION

Let us now consider the first-order approximation system:

$$dx_{n+1}^0(t)/dt = 0.5[(A_1^0 + B_1^0)x_{n+1}^0(t)$$

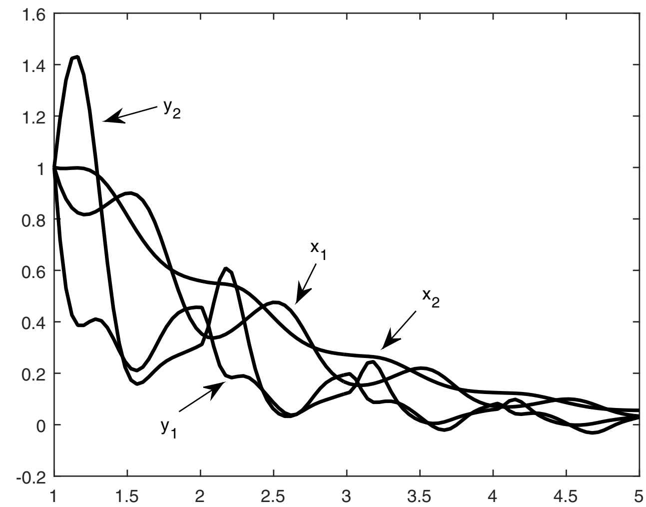

Fig. 1. Plot of the solution of system (11) for $\delta_1 = 0.6$, $\delta_2 = 0.2$.

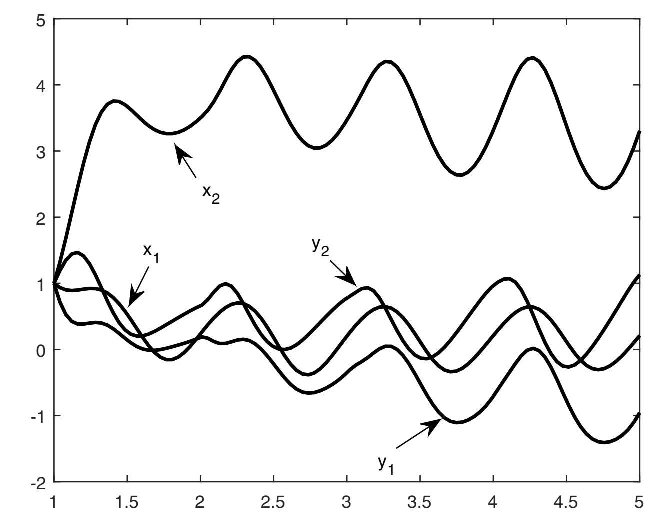

Fig. 2. Plot of the solution of system (11) for $\delta_1 = 8$, $\delta_2 = 0.2$.

$$\begin{aligned} &+A_2^0 y_{n+1}^0(t) + B_2^0 y_n^0(t)] + C_1 v_n \\ &\varepsilon_n dy_{n+1}^0(t)/dt = e^t[(A_3(t) + B_3(t))x_{n+1}^0(t) \\ &\quad +A_4(t)y_{n+1}^0(t) + B_4(t)y_n^0(t) + C_2 w_n], \qquad (12) \end{aligned}$$

where $\{v_n, w_n\}^\top$ is the control vector. In view of estimate (10) we have from the first subsystem the asymptotic equality for large enough n

$$x_{n+1}^0(t) \approx -(A_1^0 + B_1^0)^{-1}\left(A_2^0 y_{n+1}^0(t) + B_2^0 y_n^0(t) + 2C_1 v_n\right).$$

Substitute this expression into the second subsystem in the first approach system. We obtain a control system for which sufficient exponential stability conditions were formulated in the previous section

$$\begin{aligned} &\varepsilon_n dy_{n+1}^0(t)/dt = e^t(-(A_3(t) + B_3(t))(A_1^0 + B_1^0)^{-1} \\ &\quad \cdot(A_2^0 y_{n+1}^0(t) + B_2^0 y_n^0(t) + 2C_1 v_n) \\ &\quad +A_4(t)y_{n+1}^0(t) + B_4(t)y_n^0(t) + C_2 w_n). \qquad (13) \end{aligned}$$

By reducing the terms in the right-hand side of equality (13) and assuming $w_n \equiv 0$, we will stabilize system (13) in two steps. In the beginning, we stabilize the system without delayed members

$$\varepsilon_n dy^0_{n+1}(t)/dt = e^t \left(-(A_3(t)+B_3(t))(A^0_1+B^0_1)^{-1} \cdot[A^0_2 y^0_{n+1}(t) + 2C_1 v^1_n] + A_4(t)y^0_{n+1}(t)\right) = e^t \left(H(t)y^0_{n+1}(t) + \bar{C}_1 v^1_n\right), \tag{14}$$

where v^1_n is the control vector.

Let for $v^1_n \equiv 0$ the solution of system (14) be unstable, or stable, but not asymptotically. In order to effectively use the sufficient conditions of stability (8)-(9), consider the behavior of the eigenvalues $\lambda_H(t)$ of the matrix $H(t)$. Obviously, condition (8) is not satisfied (otherwise the case would have been asymptotic stability). Then there are positive constants $\bar{\gamma}_1$, $\bar{\gamma}_2$:

$$-\bar{\gamma}_1 \le Re(\lambda_H(t)) \le \bar{\gamma}_2. \tag{15}$$

Taking into account the auxiliary system (14) with the "frozen" coefficients and (15) we obtain

$$d\hat{y}^j(t)/dt = (H(t_j) + \bar{\gamma}_2 E)\hat{y}^j(t) + \bar{C}_1 \hat{v}_j(t_j),$$
$$t_{j+1} = t_j + h, \;\; h = \frac{\tau}{2k+1}, \;\; j = 0,1,...,2k+1, \tag{16}$$

Here E is the identity $m \times m$ matrix, $2k+1$ is the number of partitions of τ. It is obvious that some of these systems are unstable at some t_j. We will stabilize the system (16) in accordance with the rule given in (Furasov, 1977, p. 97), that is

$$\hat{v}_j(t_j) = -\bar{C}^\top_1 \Gamma_j \hat{y}^j(t_j). \tag{17}$$

Here Γ_j are symmetric matrices of the sizes $m \times m$ that satisfy nonlinear equations (Furasov, 1977, p. 97)

$$\Gamma_j[H(t_j) + \bar{\beta}_2 E] + [H(t_j) + \bar{\beta}_2 E]^\top \; \Gamma_j - 2\Gamma_j \bar{C}_1 \bar{C}^\top_1 \Gamma_j = -\alpha\Gamma_j + \delta E, \tag{18}$$

where δ is a small positive constant, α, $\bar{\beta}_2$ are positive constants that we can control. Equation (18) is solvable if the rank of the matrix

$$\{\bar{C}_1, [H(t) + \bar{\beta}_2 E]\bar{C}_1, [H(t) + \bar{\beta}_2 E]^2 \bar{C}_1, ..., [H(t) + \bar{\beta}_2 E]^{m-1}\bar{C}_1\}, \;\; t \in [0, \tau]$$

is equal to m. By solving equation (18) we obtain, according to (17), the corrected matrices $H_s(t_j) = H(t_j) + \bar{\gamma}_2 E - \bar{C}_1 \bar{C}^\top_1 \Gamma_j$ which are the "stabilized" matrix $H_s(t_j)$ at the points $t_j \in [0, \tau]$. At the points where $Re(\lambda_H(t_j)) < -\bar{\gamma}_2$ we assume $\hat{v}_j \equiv 0$. Thus we obtain a set of values $H_s(t_j)$ of the periodic matrix $H_s(t)$, obtained as a result of stabilization of the system with frozen coefficients. But then, as follows from (Fikhtengolts, 2002, p. 478), we can always choose a set of coefficients $\alpha^s_0, \alpha^s_1, ..., \alpha^s_k; \beta^s_1, \beta^s_2, ..., \beta^s_k$ of trigonometric polynomial of k-th order

$$H_k(t) = \frac{\alpha^s_0}{2} + \sum_{j=1}^{k} \alpha^s_j \cos\left(\frac{2\pi jt}{\tau}\right) + \sum_{j=1}^{k} \beta^s_j \sin\left(\frac{2\pi jt}{\tau}\right),$$

the values of which at $t = t_j$ are $-\bar{C}^\top_1 \Gamma(t_j)$. The following relations are valid

$$\alpha^s_0 = \frac{2}{2k+1}\sum_{j=1}^{2k} H_k(jh), \;\; \alpha^s_i = \frac{2}{2k+1}\sum_{j=1}^{2k} H_k(jh)\cos(\omega_{ij}),$$
$$\beta^s_j = \frac{2}{2k+1}\sum_{j=1}^{2k} H_k(jh)\sin(\omega_{ij}), \;\; \omega_{ij} = \frac{2\pi i}{\tau} jh, \;\; 1 \le i \le k. \tag{19}$$

Because of our assumptions, the matrix $H(t)$ is differentiable sufficient number of times, hence the "corrected" matrix in (19) will has a bounded variation. Then the following limits are valid

$$\lim_{k\to\infty} \alpha^s_0 = -\frac{2}{\tau}\int_0^\tau \bar{C}^\top_1 \Gamma(\zeta) d\zeta,$$
$$\lim_{k\to\infty} \alpha^s_i = -\frac{2}{\tau}\int_0^\tau \bar{C}^\top_1 \Gamma(\zeta) \cos\left(\frac{2\pi i}{\tau}\zeta\right) d\zeta,$$
$$\lim_{k\to\infty} \beta^s_i = -\frac{2}{\tau}\int_0^\tau \bar{C}^\top_1 \Gamma(\zeta) \sin\left(\frac{2\pi i}{\tau}\zeta\right) d\zeta.$$

Obviously, for sufficiently large k the trigonometric polynomial $H_k(t)$ close enough approximates the vector function $-\bar{C}^\top_1 \Gamma(t)$. The inequality $\|H_k(t) - H_{k-1}(t)\| < \bar{\varepsilon}$, where $\bar{\varepsilon}$ is a small enough positive number may be a criterion for quality of this approximation. Now suppose the control in system (14) is equal to $v^1_n(t) = H_k(t)\hat{y}^0_{n+1}(t)$, we obtain that with sufficiently large k eigenvalues of $\lambda_{H_k}(t)$ of corrected ("stabilized") matrices $H_k(t) \approx H_s(t)$ will satisfy the estimate

$$Re(\lambda_{H_k}(t)) \le -\frac{\bar{\gamma}_2}{2}. \tag{20}$$

Note that due to the fact that we can choose the value $\alpha > 0$, in some cases we can effectively influence the value of $Re(\lambda_{H_k}(t))$ (Furasov, 1977, p. 97).

Consider now the control system

$$\varepsilon_n dy^0_{n+1}(t)/dt = e^t[-(A_3(t)+B_3(t))(A^0_1+B^0_1)^{-1} \cdot(A^0_2 y^0_{n+1}(t) + B^0_2 y^0_n(t) - 2C_1 H_k(t) y_{n+1}(t) + 2C_1 v^2_n) + A_4(t)y^0_{n+1}(t) + B_4(t)y^0_n(t)] = e^t[H_s(t)y^0_{n+1}(t) + \bar{B}(t)y^0_n(t) + \bar{C}_1 v^2_n]. \tag{21}$$

Let $v^2_n \equiv 0$. The sufficient condition for asymptotic stability of system (21) is the exponential stability of the difference system

$$y^0_{n+1}(t) = -(H_s(t))^{-1}\bar{B}(t)y^0_n(t),$$

which is achieved, for example, when the following inequality is satisfied (Halanay, 1971, p. 69)

$$|\rho_H(t)| < \sigma, \;\; \sigma = const, \;\; 0 < \sigma < 1, \tag{22}$$

where $\rho_H(t)$ is the vector of the eigenvalues of the matrix $-H^{-1}_s(t)\bar{B}(t)$. Sometimes (by selecting sufficiently large $\alpha > 0$), inequality (22) is satisfied, that is system (21) is exponentially stable due to the fact that $\alpha \to \infty$ in (18). If this system is unstable or stable, but not asymptotically for $v^2_n \equiv 0$, further stabilization is needed. Namely, stabilization of the difference system

$$H_s(t)y^0_{n+1}(t) = -H_s(t)^{-1}\bar{B}(t)y^0_n(t) - H_s(t)^{-1}\bar{C}_1 v^2_n. \tag{23}$$

We will stabilize system (23) by a method similar to the one above. Consider auxiliary control system

$$z_{n+1}(t) = -\frac{2}{\bar{l}}\left[H^{-1}_s(t)\bar{B}(t)z_n(t) + H^{-1}_s(t)\bar{C}_1\bar{v}_n(t)\right],$$
$$\bar{l} = const, \;\; 0 < \bar{l} < 1. \tag{24}$$

We assume that for a given constant $\bar{l}$ the rank of the matrix

$$\left\{\frac{-2}{\bar{l}}H^{-1}_s(t)\bar{C}_1, \frac{4}{\bar{l}^2}H^{-1}_s(t)\bar{B}(t)H^{-1}_s(t)\bar{C}_1, ...,\right.$$

$$\frac{(-2)^m}{\bar{l}^m}\left(H_s^{-1}(t)\bar{B}(t)\right)^{m-1}H_s^{-1}(t)\bar{C}_1\Big\}$$

is equal to m, i.e. system (24) is completely controllable. We will stabilize the auxiliary difference systems

$$z_{n+1}^j(t_j) = -\frac{2}{\bar{l}}\left[H_s^{-1}(t_j)\bar{B}(t_j)z_n^j(t_j) + H_s^{-1}(t_j)\bar{C}_1 v_j)\right],$$

$$t_1 = h,\ t_{j+1} = t_j + h,\ h = \frac{\tau}{2\bar{k}+1}, \quad (25)$$

by minimizing the functional

$$Q_j = \sum_{i=0}^{\infty}(z_i^j)^\top(t_j)Gz_i^j(t_j) + \bar{v}_i^\top(t_j)R\bar{v}_i(t_j)$$

for each system. (G, R are positive-definite matrices of sizes respectively $m\times m$ and $r\times r$.) It is known (Grebenshchikov (1992)) that the required controls have the form

$$\bar{v}_j = -\frac{2}{\bar{l}}\left(R + \hat{C}_1^\top(t_j)P(t_j)\hat{C}_1(t_j)\right)^{-1}$$

$$\cdot\hat{C}_1^\top(t_j)P(t_j)H_s^{-1}(t_j)\bar{B}(t_j)z_n^j(t_j) = \hat{B}_v(t_j)z_n^j(t_j),$$

$\hat{C}_1(t_j) = -2(\bar{l}H_s)^{-1}(t_j)\bar{C}_1$; $P(t_j)$ are positive definite, symmetric matrices of dimension $r\times r$, which satisfy the matrix equations

$$\left(\frac{4}{\bar{l}^2}H_s^{-1}(t_j)\bar{B}(t_j)\right)^\top P(t_j)H_s^{-1}(t_j)\bar{B}(t_j)$$

$$-P(t_j) + G - \frac{2}{\bar{l}}\hat{C}_1^\top(t_j)P(t_j)\hat{C}_1(t_j) \quad (26)$$

$$\cdot\left(R + \hat{C}_1^\top(t_j)P(t_j)\hat{C}_1(t_j)\right)^{-1}\hat{C}_1(t_j)P(t_j)H_s^{-1}(t_j)\bar{B}(t_j) = 0$$

(equations (26) are solvable due to the full controllability of system (25)).

Again we get a set of $\hat{B}_v(t_j)$, $j = 1,\ldots,2\bar{k}+1$ and also $-(2\bar{l}H_s(t_j))^{-1}\cdot\left[\bar{B}(t_j) + \bar{C}_1\hat{B}_v(t_j)\right]$. Note that at the points $\bar{t}_j$, where $|\rho_H(-H_s^{-1}\bar{B}(\bar{t}_j))| < 0.5\bar{l}$ we suppose $\hat{B}_v(\bar{t}_j) = 0$. Now (in the same way we found the control in (14)) consider the trigonometric polynomial of $\bar{k}$-th order

$$\bar{H}_{\bar{k}}(t) = \frac{\bar{\alpha}_0^s}{2} + \sum_{j=1}^{\bar{k}}\bar{\alpha}_j^s\cos(\frac{2\pi jt}{\tau}) + \sum_{j=1}^{\bar{k}}\bar{\beta}_j^s\sin(\frac{2\pi jt}{\tau}),$$

the values of which at $t = t_j$ are $\hat{B}_v(t_j)$. Assuming

$$\hat{B}_v(t) \approx \bar{H}_{\bar{k}}(t) = \frac{\bar{\alpha}_0}{2} + \sum_{j=1}^{\bar{k}}\bar{\alpha}_j\cos\frac{2\pi j}{\tau}t + \sum_{j=1}^{\bar{k}}\bar{\beta}_j\sin\frac{2\pi j}{\tau}t,$$

we obtain that the eigenvalues ρ_H^s of the stabilized matrix $(-H_s)^{-1}[\bar{B}(t) + \bar{C}_1\bar{H}_{\bar{k}}(t)]$ for large enough $\bar{k}$ satisfy the inequality

$$|\rho_H^s| < \bar{l} < 1. \quad (27)$$

Let us now consider system (13). Taking into account (25), with the chosen control $v_n = \hat{H}_k(t)y_{n+1}^0(t) + \bar{H}_{\bar{k}}(t)y_n^0(t)$ we get the system

$$\varepsilon_n dy_{n+1}^0(t)/dt = e^t\{[H_1(t) + \bar{C}_1\hat{H}_k(t)]y_{n+1}^0(t)$$

$$+[\bar{B}(t) + \bar{C}_1\bar{H}_{\bar{k}}(t)]y_n^0(t)\}. \quad (28)$$

The eigenvalues of the matrix $H_1(t) + \bar{C}_1\hat{H}_k(t)$ and matrix $[H_1(t) + \bar{C}_1\hat{H}_k(t)]^{-1}[\bar{B}(t) + \bar{C}_1\bar{H}_{\bar{k}}(t)]$ satisfy estimates, respectively (20) and (27). From the first section, for solution of system (28) the following inequality holds

$$\|y_{n+1}^0(t)\|_\tau \le M_0(q_0)^n\sup_\xi\|\phi(\xi)\|,$$

$$M_0 = const,\ M_0 > 1,\ q_0 = const,\ 0 < q_0 < 1, \quad (29)$$

or on an infinite interval we have an estimate

$$\|y^0(t)\| \le \bar{M}_1 e^{-\beta_4(t-T)}\sup_{T-\tau\le t\le T}\|y^0(t)\|,$$

$$M_1 = const,\ M_1 > 1,\ \beta_4 = \frac{-\ln(q_0)}{\tau},\ t \ge T.$$

Considering (4), the solution $x^0(t)$ is representable in an integral form

$$x^0(t) = e^{0.5(A_1+A_2)(t-T)}x^0(T)$$

$$+\int_T^t e^{0.5(A_1+A_2)(t-s)}(A_2^0y^0(s) + B_2^0y^0(s-\tau))ds,\ t \ge T$$

(T is a sufficiently large positive number). Hence, in view of (7) and (29), we obtain the inequality

$$\|x^0(t)\| \le M_2e^{-\beta_2(t-T)}\|x^0(T)\| + M_2M_1\int_T^t e^{-\beta_2(t-s)}$$

$$\cdot\left(\|A_2^0\|e^{-\beta_4(s-T)} + \|B_2\|e^{-\beta_4(s-T-\tau)}\right)ds\sup_{T-\tau\le t\le T}\|y^0(t)\|,$$

$$M_2 = const,\ M_2 > 1,\ t \ge T.$$

The first term in the right-hand side of this inequality tends to zero when $t\to\infty$. Without loss of generality we consider that $\beta_4 < \beta_2$. But then we have the following assessment for the integral term on the right-hand side of the last inequality

$$\int_T^t e^{-\beta_2(t-s)}\left(\|A_2^0\|e^{-\beta_4(s-T)} + \|B_2\|\right.$$

$$\left.\cdot e^{-\beta_4(s-T-\tau)}\right)ds\cdot\sup_{T-\tau\le t\le T}\|y^0(t)\| = \mathbf{O}\left(e^{-\beta_4(t)}\right).$$

The first-order approximation system (12) is exponentially stable. But then for the control $u(t) = \hat{H}_k(t)y(t) + \bar{H}_{\bar{k}}(t)y(t-\tau)$ original system (1) is exponentially stable. Note that in the case of instability of the first subsystem in (2) the problem is reduced to stabilization of the system without delay and with constant coefficients (Letov's problem). In the case of instability of the second subsystem in (13) stabilization is carried out with the control w_n according to the algorithm described above.

REFERENCES

E.A. Barbashin *Introduction to the theory of stability.* Nauka, Moscow, 1967. (in russian)

R. Bellman and K. Cooke *Differential-difference equations.* Academic Press, New York, 1963.

G.M. Fikhtengolts *Differential and integral calculus. Vol. 3.* Nauka, Moscow, 2002. (in russian)

L. Fox, D.F. Mayers, J.R. Ockendon and A.B. Tayler On a functional differential equation. *J. Inst. Math. Applics.*, 8:271–307, 1971.

V.D. Furasov *Stability of motion, estimates and stabilization.* Nauka, Moscow, 1977. (in russian)

B.G. Grebenshchikov On the stabilization of stationary linear systems with time-dependent delay. *Izvestiya Vuzov. Matematika. Dep. in VINITI*, N 438492, 1992. (in russian)

B.G. Grebenshchikov The first-approximation stability of a nonstationary system with delay. *Russian Mathematics*, 56(2):29–36, 2012.

B.G. Grebenshchikov and A.B. Lozhnikov Stability and stabilization of one class of linear nonstationary constant-delay systems. *Journal of Computer and Systems Sciences International*, 56(2):173–185, 2017.

A. Halanay and D. Wexler *Qualitative theory of pulse systems*. Mir, Moscow, 1971.

Yu.A. Mitropolskii *The method of averaging in nonlinear mechanics*. Naukova Dumka, Kiev, 1971. (in russian)

J.R. Ockendon and A.B. Tayler The dynamics of a current collection system for an electric locomotive *Proceedings of the Royal Society of London A*, 322:447–468, 1971.